A new Dirac-type equation for tachyonic neutrinos

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Based on experimental evidences supporting the hypothesis that neutrinos might be tachyonic fermions, a new Dirac-type equation is proposed and a spin-$\frac{1}{2}$ tachyonic quantum theory is developed. The new Dirac-type equation provides a solution for the puzzle of negative mass-square of neutrinos. This equation can be written in two spinor equations coupled together via nonzero mass while respecting the maximum parity violation, and it reduces to one Weyl equation in the massless limit. Some peculiar features of tachyonic neutrino are discussed in this theoretical framework.

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I. INTRODUCTION

A model has recently been presented to fit the cosmic ray spectrum at $E \approx 1-4$ PeV [1-3] using the hypothesis that the electron neutrino is a tachyon. This model yields a value for $m^2(\nu_e) \approx -3eV^2$, which is consistent with the results from recent measurements in tritium beta decay experiments [4-6]. Moreover, the muon neutrino also exhibits a negative mass-square [7]. However, up to now, there is no satisfactory tachyonic quantum theory to describe neutrinos as spin-$\frac{1}{2}$ fermions.

The negative value of the neutrino mass-square simply means:

$$E^2/c^2 - p^2 = m_\nu^2 c^2 < 0 \tag{1}$$

The right-hand side in Eq.(1) can be rewritten as $(-m_s^2 c^2)$, then m_s has a positive value.

Based on special relativity and known as re-interpretation rule, tachyon as a hypothetical particle was proposed by Bilaniuk et al. in the Sixties [8-10]. For tachyons, the relation of momentum and energy is shown in Eq.(1). The negative value on the right- hand side of Eq.(1) means that p^2 is greater than $(E/c)^2$. The velocity of a tachyons, u_s, is greater than speed of light. The momentum and energy in terms of u_s are as follows:

$$p = \frac{m_s u_s}{\sqrt{u_s^2/c^2 - 1}}, \quad E = \frac{m_s c^2}{\sqrt{u_s^2/c^2 - 1}} \tag{2}$$

where the subscript s means superluminal particle, i.e. tachyon.

Any physical reference system is built by subluminal particles (such as atoms, molecules etc.), which requires that a reference frame must move slower than light. On the other hand, once a tachyon is created in an interaction, its speed is always greater than the speed of light. Neutrino is the most possible candidate for a tachyon because it has left-handed spin in any reference frame [11,12]. However, anti-neutrino always has right-handed spin. Considering the measured mass-square is negative for the muon neutrino, Chodos, Hauser and Kostelecky [11] suggested in 1985 that the muon neutrino might be a tachyon. They also suggested that one could test the tachyonic neutrino in high energy region using a strange feature of tachyon: E_ν could be negative in some reference frames [13,14]. This feature has been further studied by

Ehrlich [1-3]. Therefore, it is required to construct a tachyonic quantum theory for neutrinos.

The first step in this direction is usually to introduce an imaginary mass, but these efforts could not reach a point for constructing a consistent quantum theory. Some early investigations of a Dirac-type equation for tachyonic fermions are listed in Ref.[15]. An alternative approach was investigated by Chodos et al. [11]. They examined the possibility that muon neutrino might be tachyonic fermion. A form of the lagrangian density for tachyonic neutrinos was proposed. Although they did not obtain a satisfatory quantum theory for tachyonic fermions, they suggested that more theoretical work would be needed to determine physically acceptable theory.

2. A NEW DIRAC-TYPE EQUATION

In this paper, we will start with a different approach to derive a new Dirac-type equation for tachyonic neutrinos. In order to avoid introducing imaginary mass, Eq. (1) can be rewritten as

$$E = (c^2p^2 - m_s^2c^4)^{1/2} \tag{3}$$

where m_s is called proper mass. For instance, $m_s(\nu_e)$= 1.6 eV, if taking $m^2(\nu_e) = -2.5eV^2$ [16]. We follow Dirac's approach [17], Hamiltonian must be first order in momentum operator $\hat{p}$:

$$\hat{E} = c\vec{\alpha} \cdot \hat{p} + \beta_s m_s c^2 \tag{4}$$

with ($\hat{E} = i\hbar\partial/\partial t, \hat{p} = -i\hbar\nabla$). $\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ and β_s are 4× 4 matrix, which are defined as

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta_s = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \tag{5}$$

where σ_i is 2×2 Pauli matrix, I is 2×2 unit matrix. Notice that β_s is a new matrix, which is different from the one in the traditional Dirac equation. We will discuss the property of β_s in a later section.

When we take square for both sides in Eq.(4), and consider the following relations:

$$\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij}$$

$$\alpha_i\beta_s + \beta_s\alpha_i = 0$$

$$\beta_s^2 = -1 \tag{6}$$

then the Klein-Gordon equation is derived, and the relation in Eq.(1) or Eq. (3) is reproduced. Since Eq.(3) is related to Eq. (2), this means β_s is a right choice to describe neutrinos as tachyons.

We now study the spin-$\frac{1}{2}$ property of neutrino (or anti-neutrino) as a tachyonic fermion.

Denote the wave function as

$$\Psi = \begin{pmatrix} \varphi(\vec{x}, t) \\ \chi(\vec{x}, t) \end{pmatrix} \quad with \quad \varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

the complete form of the new Dirac-type equation, Eq.(4), becomes

$$\hat{E}\Psi = c(\vec{\alpha} \cdot \hat{p})\Psi + \beta_s m_s c^2 \Psi \tag{7}$$

It can also be rewritten as a pair of two-component equations:

$$\begin{aligned} i\hbar \frac{\partial \varphi}{\partial t} &= -ic\hbar \vec{\sigma} \cdot \nabla \chi + m_s c^2 \chi \\ i\hbar \frac{\partial \chi}{\partial t} &= -ic\hbar \vec{\sigma} \cdot \nabla \varphi - m_s c^2 \varphi \end{aligned} \tag{8}$$

From the equation (8), the continuity equation is derived:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0 \tag{9}$$

and we have

$$\rho = \varphi^\dagger \chi + \chi^\dagger \varphi, \quad \vec{j} = c(\varphi^\dagger \vec{\sigma} \varphi + \chi^\dagger \vec{\sigma} \chi) \tag{10}$$

where ρ and $\vec{j}$ are probability density and current; $\varphi^\dagger$ and $\chi^\dagger$ are the Hermitian adjoint of φ and χ respectively.

Eq.(10) can be rewritten as

$$\rho = \Psi^\dagger \gamma_5 \Psi, \quad \vec{j} = c(\Psi^\dagger \vec{\Sigma} \Psi) \tag{11}$$

where γ_5 and $\vec{\Sigma}$ are defined as

$$\gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \tag{12}$$

Considering a plane wave along the z axis for a right-handed particle, the helicity $H = (\vec{\sigma} \cdot \vec{p})/|\vec{p}| = 1$, then the equation (8) yields the following solution:

$$\chi = \frac{cp - m_s c^2}{E} \varphi \tag{13}$$

Indeed, there are four independent bispinors as the solutions of Eq.(8). The explicit form of four bispinors are listed in the Appendix.

Let us now discuss the property of matrix β_s in Eq.(5). Notice that it is not a 4×4 hermitian matrix. As we know, a non-hermitian Hamiltoian is not allowed for a subluminal particle, but it does work for tachyonic neutrinos. Moreover, the relation between the matrix β_s and the traditional matrix β is as follows:

$$\beta_s = \beta \gamma_5 \quad where \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{14}$$

III. PARITY VIOLATION FOR NEUTRINOS

In order to compare the new Dirac-type equation Eq.(7) with the two component Weyl equation in the massless limit, we now consider a linear combination of φ and χ :

$$\xi = \frac{1}{\sqrt{2}}(\varphi + \chi), \quad \eta = \frac{1}{\sqrt{2}}(\varphi - \chi) \tag{15}$$

where $\xi(\vec{x}, t)$ and $\eta(\vec{x}, t)$ are two-component spinor functions. In terms of ξ and η, Eq.(10) becomes

$$\rho = \xi^\dagger \xi - \eta^\dagger \eta, \quad \vec{j} = c(\xi^\dagger \vec{\sigma} \xi + \eta^\dagger \vec{\sigma} \eta) \tag{16}$$

In terms of Eq.(15), Eq.(8) can be rewritten in Weyl representation:

$$i\hbar \frac{\partial \xi}{\partial t} = -ic\hbar \vec{\sigma} \cdot \nabla \xi - m_s c^2 \eta$$

$$i\hbar \frac{\partial \eta}{\partial t} = ic\hbar \vec{\sigma} \cdot \nabla \eta + m_s c^2 \xi \tag{17}$$

In the above equations, both ξ and η are coupled via nonzero m_s.

For comparing Eq.(17) with the well known Weyl equation, we take a limit, $m_s = 0$, then the first equation in Eq.(17) reduces to

$$\frac{\partial \xi_{\bar{\nu}}}{\partial t} = -c\vec{\sigma} \cdot \nabla \xi_{\bar{\nu}} \tag{18}$$

In addition, the second equation in Eq.(17) vanishes because $\varphi = \chi$ when $m_s = 0$.

Eq.(18) is the two-component Weyl equation for describing antineutrinos $\bar{\nu}$, which is related to the maximum parity violation discovered in 1956 by Lee and Yang [18,19]. They pointed out that no experiment had shown parity to be good symmetry for weak interaction. Now we see that, in terms of Eq.(17), once if neutrino has some mass, no matter how small it is, two equations should be coupled together via the mass term while still respecting the maximum parity violation.

Indeed, the Weyl equation (18) is only valid for antineutrinos since a neutrino always has left-handed spin, which is opposite to antineutrino. For this purpose, we now introduce a transformation:

$$\vec{\alpha} \rightarrow -\vec{\alpha} \tag{19}$$

It is easily seen that the anticommutation relations in Eq.(6) remain unchanged under this transformation. In terms of Eq.(19), Eq.(4) becomes

$$\hat{E}\Psi_\nu = -c(\vec{\alpha} \cdot \hat{p})\Psi_\nu + \beta_s m_s c^2 \Psi_\nu \tag{20}$$

Furthermore, $\vec{\sigma}$ should be replaced by $(-\vec{\sigma})$ from Eq.(5) to Eq.(18) for describing a neutrino. Therefore, the two-component Weyl equation (18) for massless neutrino becomes:

$$\frac{\partial \xi_\nu}{\partial t} = c\vec{\sigma} \cdot \nabla \xi_\nu \tag{21}$$

In fact, the transformation (19) is associated with the CPT theorem. Some related discussions can be found in Ref.[20-22]. As an application, Eq.(19) can also be used to discuss the traditional Dirac equation in terms of Ψ_d for electrons, which is

$$\hat{E}\Psi_d = c(\vec{\alpha} \cdot \hat{p})\Psi_d + \beta m_o c^2 \Psi_d \tag{22}$$

where m_o is the rest mass of electron.

Using the transformation (19), Eq.(22) becomes

$$\hat{E}\Psi_a = -c(\vec{\alpha}\cdot\hat{p})\Psi_a + \beta m_o c^2 \Psi_a \tag{23}$$

where Ψ_a means that $(-\vec{\alpha})$ is adopted in the equation. We suggest that Eq.(23) can be used to describe a positron or other subluminal antiparticles with spin-$\frac{1}{2}$.

IV. SUMMARY AND DISCUSSIONS

In this paper, The hypothesis that neutrinos might be tachyonic fermions is further investigated. A spin-$\frac{1}{2}$ tachyonic quantum theory is developed on the basis of the new Dirac-type equation. It provides a solution for the puzzle of negative mass-square of neutrinos.

The matrix β_s in the new Dirac-type equation is not a 4×4 hermitian matrix. However, based on the above study, we now realize that the violation of hermitian property is related to the violation of parity. Though a non- hermitian Hamiltoian is not allowed for a subluminal particle, it does work for tachyonic neutrinos.

As a tachyon, neutrinos have many peculiar features, which are very different from all other particles. For instance, neutrinos only have weak interactions with other particles. Neutrino has left-handed spin in any reference frame. On the other hand, anti-neutrino always has right-handed spin. This means that the speed of neutrinos must be equal to or greater than the speed of light. Otherwise, the spin direction of neutrino would be changed in some reference frames. Moreover, the energy of a tachyonic neutrino (or anti-neutrino), E_ν, could be negative in some reference frames. We will discuss the subject of the negative energy in another paper.

The electron neutrino and the muon neutrino may have different non-zero proper masses. If taking the data from Ref.[16], then we obtain $m_s(\nu_e) = 1.6eV$ and $m_s(\nu_\mu) = 0.13MeV$. In this way, we can get a natural explanation why the numbers of e-lepton and μ-lepton are conserved respectively.

According to special relativity [23], if there is a superluminal particle, it might travel backward in time. However, a re- interpretation rule has been introduced since the Sixties [8-10]. Another approach is to introduce a kinematic time under a non- standard form of the Lorentz transformation [24-28]. Therefore, special relativity can be extended to space-like region, and tachyons are allowed without causality violation.

Generally speaking, the above spin-$\frac{1}{2}$ tachyonic quantum theory provides a theoretical framework to study the hypothesis that neutrinos are tachyonic fermions. More measurements on the cosmic ray at the spectrum knee and more accurate tritium beta decay experiments are needed to further test the above theory.

APPENDIX. EXPLICIT FORM OF SOLUTIONS

For a free particle with momentum $\vec{p}$ in the z direction, the plane wave can be represented by

$$\Psi(z,t) = \psi_\sigma exp[\frac{i}{\hbar}(pz - Et)]$$

where ψ_σ is a four-component bispinor. This bispinor satisfies the wave equation (7). From Eq. (13), the explicit form of two bispinors with the positive-energy states are listed as follows:

$$\psi_1 = \psi_{\uparrow(+)} = N \begin{pmatrix} 1 \\ 0 \\ a \\ 0 \end{pmatrix}, \quad \psi_2 = \psi_{\downarrow(+)} = N \begin{pmatrix} 0 \\ -a \\ 0 \\ 1 \end{pmatrix} \qquad (A1)$$

and other two bispinors with the negative-energy states are:

$$\psi_3 = \psi_{\uparrow(-)} = N \begin{pmatrix} 1 \\ 0 \\ -a \\ 0 \end{pmatrix}, \quad \psi_4 = \psi_{\downarrow(-)} = N \begin{pmatrix} 0 \\ a \\ 0 \\ 1 \end{pmatrix} \qquad (A2)$$

where the component a and the normalization factor N are

$$a = \frac{cp - m_s c^2}{|E|}, \quad N = \sqrt{\frac{p + m_s c}{2 m_s c}} \qquad (A3)$$

For $\psi_1 = \psi_{\uparrow(+)}$, the conserved current becomes:

$$\rho = {\Psi_1}^\dagger \gamma_5 \Psi_1 = \frac{|E|}{m_s c^2}, \quad j = \frac{p}{m_s} \qquad (A4)$$

and we can also obtain a scalar:

$$\bar{\Psi}_1 \Psi_1 = \Psi_1^\dagger \beta \Psi_1 = 1 \qquad (A5)$$

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